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## MEASUREMENTS AT THE SPEED OF ULTRASOUND: FRIENDLY REGARDS FROM FERMAT

by

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### Introduction

The word “bent” has a special meaning to readers of *THE BENT*. In construction or architecture, a bent is a structural element used in post and beam construction. Its more common meaning brings to mind a shape like a rod or a line that seems headed one way but then takes a sudden turn. A common example is a stick held in water at an angle, partly immersed, partly in air. It appears bent. The explanation is familiar. The speed of light is about 33% faster in air than in water. Therefore the light rays bend (refract) when they strike the interface obliquely. This article explores how this simple concept underlies *ultrasonic* devices used in two different fields: NDT & PCI (nondestructive testing and process control instrumentation). Conversely, we’ll also try to show that devices that look pretty simple can lead one to fundamental design principles, or at least illustrate such principles in action. The device we’ll look at is an acoustic wedge, and its purpose is to bend ultrasonic rays. But first, let’s return to our stick in water.

The refracted angle is easily calculated by Snell’s Law of refraction, relating the index of refraction,  $n$ , and the sines of the angles of incidence and refraction. Suppose we designate the stick in Fig. 1 as having end points A and W. Let it be straight, one meter long, its midpoint O at the water’s surface, and angled at  $45^\circ$ . The AOW path taken by the stick is straight, “the shortest distance between two points.” The path that *light* takes from point A in air to point W in water, however, is not straight, but rather, the one that takes the shortest *time*. This is *Fermat’s principle of least time*. Finding the fastest route – the one that yields the shortest time between two points – applies to sound waves as well as to electromagnetic (light) waves. Sound waves include *ultrasonic* waves (waves having frequencies above 20 kHz). The fact that sound or ultrasound waves bend as a function of the sound

speed ratio encountered obliquely at an interface, is the basis for several devices used in industry. The particular devices may or may not be interesting to the reader at the moment. *Fermat’s principle*, however, is of considerable importance, and exploring its range of validity, and limitations, accordingly is a worthwhile pursuit. This short article does not conduct such pursuits. Instead it merely points out that even a simple device like an acoustic wedge (the analog of an optical prism, with respect to bending the incident rays) touches on important principles.

Some readers may want to explore characteristics, similarities and differences between ultrasonic and light waves. For example: why do ultrasonic waves encountering an interface obliquely, sometimes launch vibration modes different from the one comprising the initial wave? Perhaps the best-known difference between rays of light and ultrasound, is that light travels fastest in vacuum, while ultrasound doesn’t travel at all in vacuum, and in air, where it does travel, it does so much slower than in water or most solids. In glass, light exhibits dispersion, meaning light travels at different speeds for different wavelengths. *Visible* light includes wavelengths from 0.4 to 0.7  $\mu\text{m}$ . The fact that a glass prism breaks up white light into its colorful spectrum demonstrates that even over wavelengths spanning a range  $< 2$ , dispersion exists. In heavy flint glass, at an IR (infrared) wavelength of 2  $\mu\text{m}$  (beyond the red end of the visible spectrum) the index  $n = 1.6$ , while at a UV (ultraviolet) wavelength of 0.36  $\mu\text{m}$ , just beyond visible violet,  $n = 1.7$ . This 6% increase in  $n$  on going from IR to UV occurs at wavelengths differing by a factor of nearly six. Ultrasound, however, would travel in the same glass at essentially constant velocity for the same ratio of wavelengths, or even over a wider range. This can be verified at commonly used ultrasonic frequencies like 1 to 10 MHz.

What about *ether drift*? Ultrasonic anemometers utilize the fact that sound travels faster in the direction of the wind, and slower against the wind. This *contrapropagation* time-difference effect is analogous to timing identical twins swimming upstream vs downstream. The twin traveling *with* the current swims faster, relative to the shore. There is no corresponding “optical anemometer,” as is well known to anyone familiar with the Michelson-Morley experiment, or relativity ( $c = \text{constant}$ ). [1]

Before discussing the angle beam bent-ray concept and its use in NDT and PCI flow measurement, let’s take a moment to consider Fig. 1 in more detail. [Applications of Fermat’s principle of least time, or Snell’s Law of refraction, to medical diagnosis apparatus, to explaining earthquake wave arrival times in seismology, meteorology (thunder propagation) and other fields, is left to the reader’s imagination and interest.]

Figure 1 illustrates some of the consequences of speed in air being faster than in water (for light), but the opposite for ultrasound. The speeds for electromagnetic (light) waves are taken as  $c_1 = 3 \times 10^8$  m/s in air and  $c_2 = (3/1.33) \times 10^8$  m/s in water (index  $n = 1.33$ ). For acoustic or ultrasonic waves, the speeds are taken for 20°C as  $c_3 = 1482$  m/s in water and  $c_4 = 343$  m/s in air. To an observer in air, the W end of the stick looks like it is closer to the surface than it really is. Intersection point B is about 9.5 cm to the left of point O, while intersection C is about 28 cm to the right of O. The paths are bounded by

parallelogram ADWE. The acoustic path lies to the right of O and is bent more than the light path because (a)  $c_3 > c_4$ , and (b) the ratio  $c_3/c_4$  ( $1482/343 = 4.32$ ) is over three times greater than the optical index  $n$ .

A peculiar coincidence was noticed in preparing this figure. The 1972 source of  $c_3$  for pure water lists the value at 20°C as 1482.343 m/s. Water’s digits after the decimal point, 343, happen to be equal numerically to the sound speed  $c_4$  in air at the same temperature. *Another* peculiarity is that if one constructs a 1 m<sup>3</sup> box and fills it with air or other gas at standard conditions (0°C, 760 mm Hg) the *weight* of the gas in pounds very nearly equals the molecular weight  $MW$  divided by ten. Example for air:  $MW = 29$ , density = 2.9 lb/m<sup>3</sup>. Why? Hints: Avogadro’s number; gram molecular volume; 1 kg = 2.2 lb. In these hybrid units the density of water is about 2200 lb/m<sup>3</sup>.

What do (a)  $MW$  or (b) standard conditions have to do with minimizing transit time? Answer: (a) Soundspeed squared in a gas is inversely proportional to average  $MW$ . This means, if our (dry) air at 20°C were to pick up some moisture, its (dry)  $MW$  of 29 would be diluted with some H<sub>2</sub>O of  $MW = 18$ , and the speed of sound would *increase*. The increase is small,  $\leq 0.2\%$  for RH (relative humidity) = 50%, and  $\leq 0.4\%$  for RH = 100%. This dilution or averaging of  $MW$  also means the *density* of moist air is less than that of dry air at the same temperature and pressure. (b) At 0°C the speed of sound in dry air is 331 m/s, slower than at 20°C by about 3%. Now back to our story.

## Nondestructive Testing (NDT) Using Angle-Beam Transducers

About fifty years ago Moriarty [2] reported that a plastic wedge, Fig. 2a, in which *longitudinal* waves were introduced obliquely against a steel pipe, provided a useful way to generate *shear waves in the pipe* for purposes of inspection. The shear waves, zigzagging down the pipe wall at angles of incidence in the pipe that did *not* mode convert, were able to travel substantial distances along the pipe. If the pipe geometry is uniform, then reflections occur only when a defect or other discontinuity is encountered. The angle beam wedge is used routinely nowadays to inspect welds, and to find cracks or disbands. Depending on the wedge angle, one can launch shear or Rayleigh (surface) or Lamb (plate) waves in the

metal object. The refracted angle for Rayleigh or Lamb waves can be 90°, because the speed of these waves can be roughly 50% greater than the speed of the compressional (longitudinal) wave in the wedge. The ultrasonic inspection frequencies are typically 1 to 10 MHz. However, if very tiny flaws are to be detected, a shorter wavelength is required (like going from an optical microscope to an electron beam microscope, for higher resolution). If the required short wavelength is not 1 mm but rather 1 μm, the ultrasonic frequency must be raised by a factor of 1000.

## Clamp-On Ultrasonic Flowmeter Transducer

In the industrial field of process control, flow is one of the four most important measurands. (The others are temperature, pressure, and level.) The first clamp-on ultrasonic flowmeters appear to have originated in Japan. By 1964 it had been recognized by the Japanese clamp-on flowmeter pioneers Yamamoto and co-workers that a plastic wedge much like the axially-oriented one in Fig. 2a could be used to launch waves through a steel pipe wall, into and across the water inside, and to receive the waves at a diagonally opposite point or after one bounce (Fig. 2b). By timing the ultrasonic waves in each direction, the small difference in transit times,  $\Delta t$ , can be interpreted in terms of the average flow velocity across the tilted-diameter path. This method has since been developed by several manufacturers and used in tens of thousands of applications.

At temperature extremes, either at the cold cryogenic domain of liquid nitrogen, or at the hot end as exemplified by superheated water or superheated steam, the plastic wedge design of Fig. 2b no longer suffices, for two reasons. First, the piezoceramic probably would disbond from the plastic wedge, due to differential thermal expansion/contraction. Second, at high temperature, the piezoceramic would no longer generate and detect ultrasonic waves, analogous to a magnet becoming demagnetized. While these problems can be overcome through judicious choice of materials, that route is expensive and does not necessarily cover all temperatures of interest.

A different solution is represented in Fig. 3. This solution adds a buffer waveguide to the wedge. To avoid unwanted mode conversions along the

waveguide, and to introduce the incident wave at as low a sound speed as is currently practical, the piezoceramic element is selected to be in the shear mode. This means particle motion is transverse to the direction of propagation. This device has now been used in about a hundred different flow applications at high temperature, and also at cryogenic temperature in Korea and Japan. The ultrasonic wave is “bent” in the fluid and travels just a few degrees off normal, but this path is angled enough to produce a time difference between upstream and downstream interrogation directions. (In the fluid the ultrasonic wave is not only refracted but also mode converted to the longitudinal mode. Inviscid fluids do not support shear waves.) We leave as “exercises for the reader,” analysis of how the energy is partitioned among the various modes; how the average velocity along the tilted diameter relates to the true average over the pipe’s cross-section; how the individual waves or pulses are timed to subnanosecond precision. The author trusts that such aspects will not leave the reader frustrated and “bent out of shape.” Some hints are found in [3-6] and in the references therein. Information on limitations of Snell’s Law may be found in [7].

## Want More Details?

For detailed information on the use of the OKS transducer of Fig. 3 on 3-inch pipes conveying water at 282°C, please check PCI R&D pages for the following report: Burnham, R. N., McCarey, R. A., Khrakovsky, O. A., and Lynnworth, L. C., Measurement of the Flow of Superheated Water in Blowdown Pipes at MP2 Using an Ultrasonic Clamp-On Method (2001).

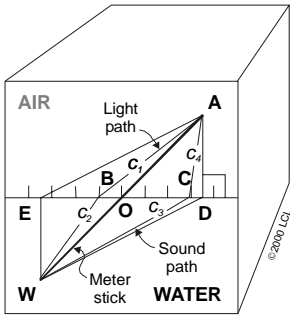
## Conclusion

Nondestructive testing of metal plates and pipes, and noninvasively measuring the flow of fluids through pipes, are of concern to engineers in two apparently diverse fields. The acoustic wedge, however, forms part of the solution for each of these two problems, and so forms a bridge between two fields and between the engineers in those fields. Fermat's principle of least time, well known in optics, also forms a bridge, as it applies to light and to ultrasonic waves. If one has the time and inclination to analyze a simple-looking technical device, the acoustic wedge in our

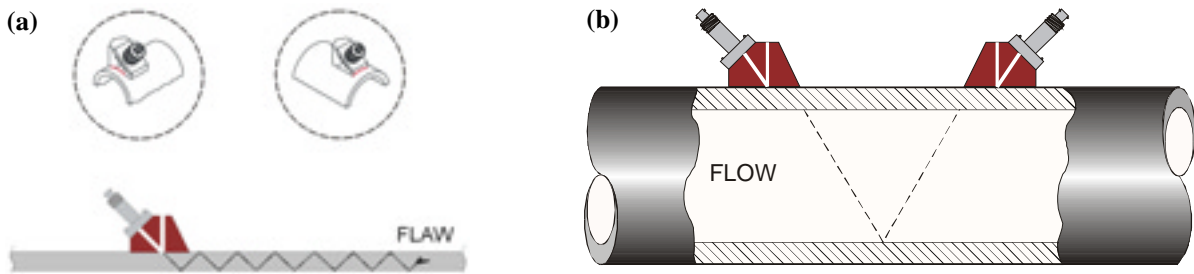
present examples, some insight is obtained into the broader field of wave propagation. Wave propagation, in turn, can yield information of value to NDT and process control engineers, to the extent the waves interact with the measurand of interest, without too much distracting influence by interfering variables.

## References

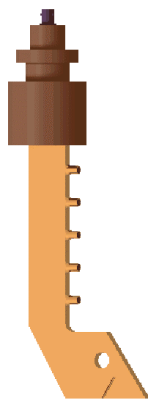
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**Figure 1.** Meter stick AOW, midpoint O, is half-immersed in water and is inclined at 45°. Paths between A and W for light rays and for ultrasound rays are drawn according to Fermat’s principle of shortest time. The lengths  $P_{\pm}$  of the longer and shorter path segments may be calculated using the law of cosines. If one denotes the distance from O to the bend at B or C as  $|x|$ , then  $P_{\pm} = (x^2 \pm 0.7071 |x| + .25)^{1/2}$ . Trying different values for  $x$ , one finds a near-minimum time for light of 3.85 ns at  $|x| = 9.5$  cm, and 1543  $\mu$ s for sound at  $|x| = 28$  cm. The time of travel along each segment  $P_{+}$  or  $P_{-}$  is obtained by dividing  $P_{\pm}$  by the appropriate speed  $c_i$ , where  $i = 1, 2, 3$  or 4.



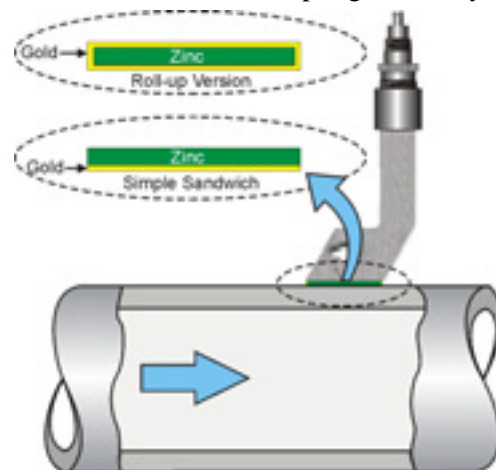
**Figure 2.** Acoustic wedges bend beams of ultrasound. The propagation of the bent or refracted beam is influenced by the scattering and transmission characteristics along its path. Ultrasonic instruments use the refracted rays to nondestructively sense flaws or noninvasively sense flow. In (a) one angle beam transducer typically is used alone in monostatic pulse-echo mode. Interrogation can be in the axial or circumferential direction. In (b) a pair of clamp-on transducers measure flow of liquid by the contrapropagation (upstream – downstream) bistatic method. If the liquid is so cold or so hot that “ordinary” plastic wedges and “ordinary” piezoceramics no longer work, one remedy is to substitute buffer waveguides having the shape of a hockey stick (Figure 3).

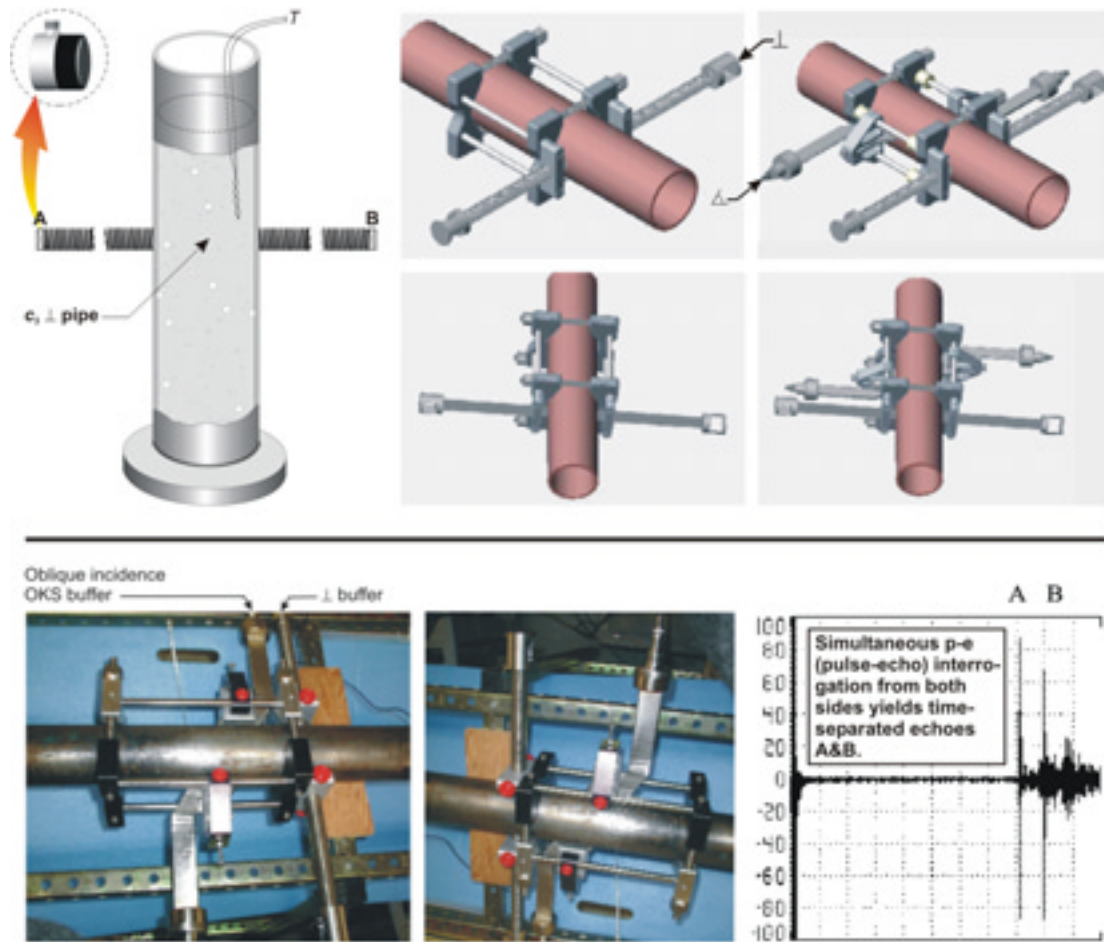


**Figure 3.** A thin solid waveguide, about 250-mm long in this example, and constructed according to U. S. Patent 6,047,602 (April 11, 2000), allows ordinary shear-mode piezoceramics to be applied to clamp-on measurement of the flow velocity of water in pipes up to approximately 300°C; in liquid hydrocarbons up to 400°C (application reported in France); and in cryogenic liquids down to approximately -200°C including conduits as small as 50 mm nominal diameter.

## What's New in the "Hockey Stick" Neighborhood Since the Above Article Appeared in *The Bent of Tau Beta Pi*, Summer 2000?

- Ultrasonic Report UR-267B, co-authored by customer Robert Burnham, covering two years' use of OKS transducers at the Millstone Nuclear Power Generating Plant in Connecticut, was released and appears elsewhere on this website. (Pipe size and material: 3-inch schedule 40, carbon steel.)
- Two high-temperature *temporary couplants* are now available for short-term evaluations (~hour, or days). These are, respectively, an anti-seize compound, and an elastomer.
- For permanent coupling (years) the standard metal foil solution is a layered coupler consisting of a gold + zinc sandwich [U.S. Patent 6,349,599 (February 26, 2002)]. Gold goes against the pipe. (This patent can be viewed at [www.uspto.gov](http://www.uspto.gov)). An excerpt appears below as Figure 4.
- For applications where pipe temperature does not exceed 300°C, a 316SS OKS waveguide length of about 200 mm measured  $\perp$  pipe usually suffices.
- An easier-to-use *clamp* was derived in January 2002 from the precision GC868 clamp. (See UR-268 by Ao et al. elsewhere on this website for GC868 clamp-on *gas* flowmeter information.) This new clamp accurately aligns OKS transducers at 180° and simplifies setting or adjusting the axial spacing  $S$ . Furthermore:
- The modified CG868 clamping fixture *also* allows temporary or permanent measurements of liquid sound speed  $c_3 \perp$  pipe. (See Figure 5 below.) This  $\perp$  measurement uses longitudinal buffer rods. By *starting* with  $c_3$ , one (a) qualifies the location and liquid; (b) checks wall thickness; (c) calculates  $S$  based on the *measured*  $c_3$  rather than on an *estimated*  $c_3$ . This saves time and trouble on liquids of variable composition whose actual sound speed may be uncertain at the high temperature of a particular application. *Measuring  $c_3 \perp$  pipe has therefore become the recommended step 1!*
- One customer in Ohio needed to conduct high-temperature clamp-on flow surveys of one particular heat transfer liquid flowing in different pipes, mostly steel, and mostly in pipe sizes between about 2" to 8". The initial problem was the sound speed was an unknown function of temperature  $T$ . The customer did not want to use a "trial and error" method on pipes above 200°C, to find a spacing that yielded adequate signal. In the early part of 2002, this customer, first on the bench and later in his plant, measured  $c_3 \perp$  pipe from about 40 to 214°C. (This was done using the threaded buffers of Figure 5.) Having determined  $c_3$  vs  $T$ , the customer was then able to avoid guesswork and immediately and confidently set spacing  $S$  just from the pipe dimensions and wall temperature. The flowmeter automatically calculates  $S$  from  $c_3$  and the pipe dimensions, i.e. outside diameter and wall thickness.
- For liquids where  $c_3$  yields *liquid density*, the flowmeter can become a stand-alone *clamp-on mass flowmeter*. See UR-240, presented in part at the 5<sup>th</sup> ISFFM in April 2002.
- Abstract from U.S. Patent 6,349,599 that issued on February 26, 2002: An ultrasonic coupling assembly includes a barrier layer that contacts the wall, and a generally thicker layer formed of a compliant material such as zinc foil that extends over the first layer. A third layer may cover and enclose the compliant layer, e.g., by folding the first layer over and on top of the second layer to cover the edge faces thereof. An exemplary embodiment is implemented using gold foil approximately one mil [25- $\mu$ m] thick for the first layer, and zinc sheeting approximately four mils [100- $\mu$ m] thick for the second layer. Use of inert top and bottom, or outside, layers prevents the inadvertent installation of the coupling element with the compliant layer directly contacting the wall. Disk embodiments may employ a stack of separate disks formed of foils or sheets of the respective materials. For cryogenic applications, a material such as indium may be used for the second layer without risk of its alloying or migration through the first layer.





**Figure 5.** Adaptation of the clamp developed for clamp-on *gas* flow applications [see Figure 1 (d-f) in UR-240 or see UR-268], to higher-precision *liquid* clamp-on flow measurements, including  $c_3 \perp$  pipe. If the  $\perp$  buffer rods are of different lengths, e.g. 9" and 10" as in this example, simultaneous p-e (pulse-echo) interrogation from both sides yields time-separated echoes A&B. Depending on the buffer rod design, and details of the situation, it is possible to see on expanded sweeps, delayed wall ringdown patterns, and also a signal through the liquid. The waveforms in this example were obtained with an Epoch 4 NDT flaw detector and 5-MHz A108S transducers at the outer ends of the 316SS  $\phi 1$ " coarse-threaded  $\perp$  buffers. Generally speaking, one seeks the maximum usable frequency. Hence the NDT contact transducers indicated at A and B at the cool ends of the buffers can have nominal frequencies of 5, 2, and 1 MHz, tried sequentially. For an attenuating liquid, 5 MHz may be best for gaging the wall thickness, and 1 or 2 MHz, for transmitting across the liquid. At *ordinary* temperatures, the NDT transducer may be coupled directly against the pipe. In this case the same threaded "buffer" is first backed off a few turns, making room for the transducer. Then it is gently tightened against the transducer to provide the coupling force  $\perp$  pipe.